

4.3 p1

MA121

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Logarithms

Let us say we have an exponential

$$y = a^x \leftarrow a \text{ is constant, base of exponential}$$

Another way of writing it is

$$x = \log_a y$$

$$y = a^x \Leftrightarrow x = \log_a y$$

EX i) $2^3 = 8 \Leftrightarrow \log_2 8 = 3$

ii) $9^{1/2} = 3 \Leftrightarrow \log_9 3 = 1/2$

iii) $4^2 = 16 \Leftrightarrow \log_4 16 = 2$

iv) $125^{1/3} = 5 \Leftrightarrow \log_{125} 5 = 1/3$

In summary, the following are equivalents:

$$i) a^b = C \Leftrightarrow \log_a C = b$$

In a logarithm, we must have↳ the base $0 < a < 1$ or $a > 1$ ↳ the argument $C > 0$.Special case.

When the base of the logarithm is e ,
 (recall $e = \lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n \approx 2.718281 \dots$)

we call this logarithm a natural logarithm.

We write it in a shortened form.

$$\boxed{\log_e X = \ln X}$$

In a calculator, LN means $\log_e X$ LOG means $\log_{10} X$

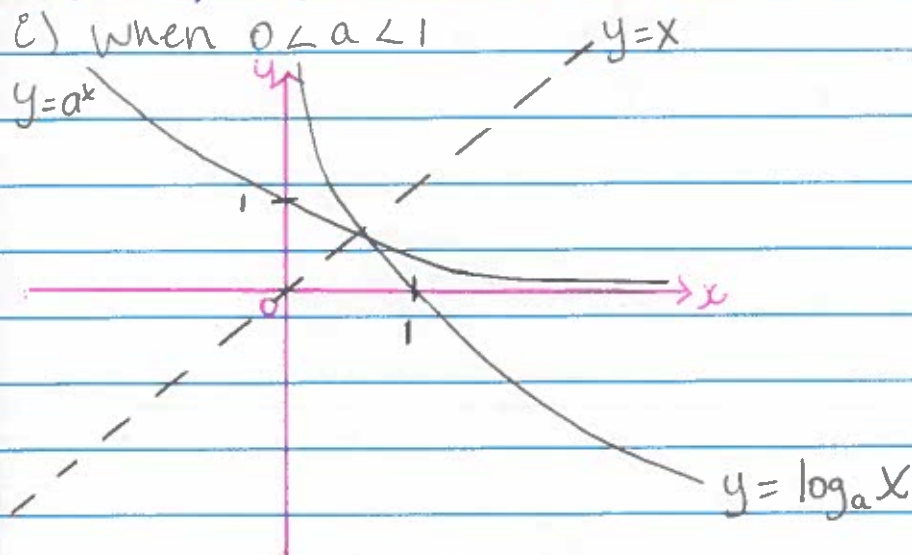
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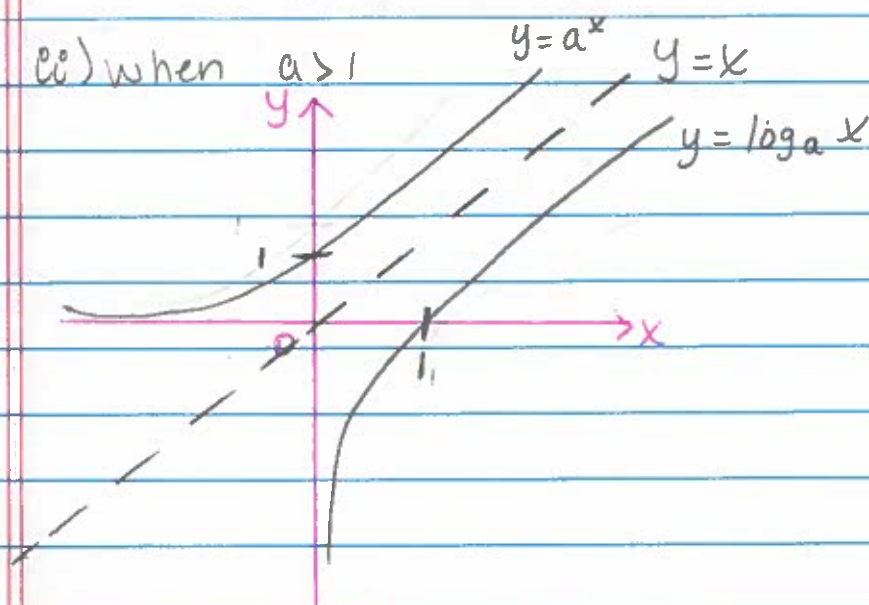
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Graphs of exponential and logarithm functions.

i) When $0 < a < 1$



ii) When $a > 1$



Properties of logarithms.

If $b > 0$, $w > 0$, $z > 0$

i) $\log_b (wz) = \log_b w + \log_b z$

ii) $\log_b \frac{w}{z} = \log_b w - \log_b z$

iii) $\log_b w^p = p \log_b w$ ← where p is any real number

iv) $\log_b b^x = x$

v) $b^{\log_b x} = x$

Corollary → i) $\log_b b = 1$ and ii) $\log_b 1 = 0$

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ex

3p 114 How long does it take for a \$500 investment to triple if the nominal rate of 1.75% per year is compounded daily?

given $P = \$500$, we want $F(t) \geq \$1500$.

given also $r = 1.75\% = 0.0175$ and $n = 365$ (times per year)

we have $F(t) = P(1 + \frac{r}{n})^{nt}$

$$\Rightarrow P(1 + \frac{r}{n})^{nt} \geq 1500$$

$$\Rightarrow 500(1 + \frac{0.0175}{365})^{365t} \geq 1500$$

$$\Rightarrow (1 + \frac{0.0175}{365})^{365t} \geq 3$$

$$\Rightarrow \ln(1 + \frac{0.0175}{365})^{365t} \geq \ln 3 \leftarrow \text{take the log of both sides}$$

$$\Rightarrow 365t \ln(1 + \frac{0.0175}{365}) \geq \ln 3$$

$$\Rightarrow t \geq \frac{\ln 3}{365 \ln(1 + \frac{0.0175}{365})}$$

$$\Rightarrow t \geq 62.77935002 \dots$$

$$\Rightarrow t \geq 62 \text{ years and } 285 \text{ days.}$$

ex.

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The base year, 1966, point base = 100.

The next year, 1967, +4.2% = $CPI(1967) = CPI(1966)[r_{eff}]^{1967-1966}$
 $= 100[1 + 0.042]^1 = 104.2$

In 1968, +5.4% = $CPI(1968) = CPI(1967)[r_{eff}]^{1968-1967}$
 $= 104.2[1 + 0.054]^1$
 $= 109.8$

In 1980, $CPI(1980) = 270.4 \leftarrow \text{calculate avg } r_{eff} \text{ from } 1966-80.$

$F(t) = P(1 + r_{eff})^t$; $r_{eff} = i$

$$\Rightarrow 270.4 = 100(1 + r_{eff})^{1980-1966}$$

$$\Rightarrow (1 + r_{eff})^{14} = \frac{270.4}{100} = 2.704$$

$$\Rightarrow 1 + r_{eff} = \sqrt[14]{2.704}$$

$$\Rightarrow r_{eff} = \sqrt[14]{2.704} - 1 \approx 0.073637 = 7.36\%$$

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Exponential Models

↳ learning curve model

"A person's mastery of a certain subject matter begins rapidly and then slows, approaching an asymptotic plateau"

Suppose L = number of potential customers.

S = number of current customers.

We want the difference to decline at an instantaneous rate, k . in essence if $L-S$ is declining, $r = -k$.

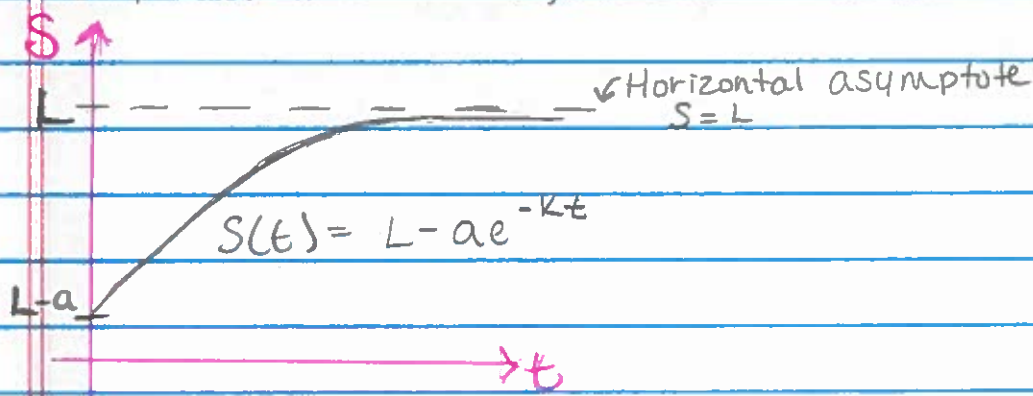
$$\text{SO, } L - S = ae^{rt}$$

$$\Rightarrow L - S = ae^{-kt}$$

$$\Rightarrow \boxed{S(t) = L - ae^{-kt}} \text{ where } t \geq 0$$

where a is a constant coefficient depending on initial sales.

> Graph of the learning curve model:



↳ logistic growth model

"In some cases, a new product does not catch on as rapidly as a learning curve model. There may be a start up time." The relative difference between the number of current customers

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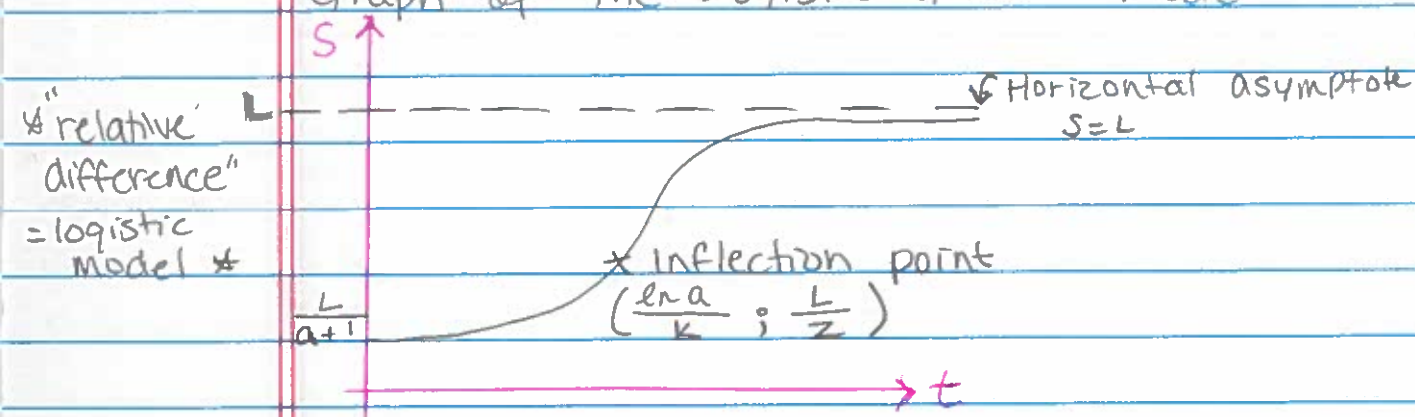
$S(t)$ and the limiting value, L , declines at an instantaneous rate, k .

$$\frac{L-S}{S} = ae^{rt} \quad \text{where } r = -k$$

$$\Rightarrow \boxed{S(t) = \frac{L}{1 + ae^{-kt}}} \quad t \geq 0$$

where a is a constant depending on initial sales.

Graph of the Logistic Growth Model



Q. How do I know which model to use?

A. Watch for the term "relative difference" in your question. This will tell you to use the logistic growth model. Otherwise, use the learning Curve model.

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ex 3p 1/4 How long does it take a parcel of land worth D dollars to double in value if it appreciates continuously at a nominal rate

